An Introduction to Decision Making

Chapter 20

GOALS

- Define the terms state of nature, event, decision alternative, and payoff.
- Organize information in a payoff table or a decision tree.
- Find the expected payoff of a decision alternative.
- Compute opportunity loss and expected opportunity loss.
- Assess the expected value of information.

Statistical Decision Theory

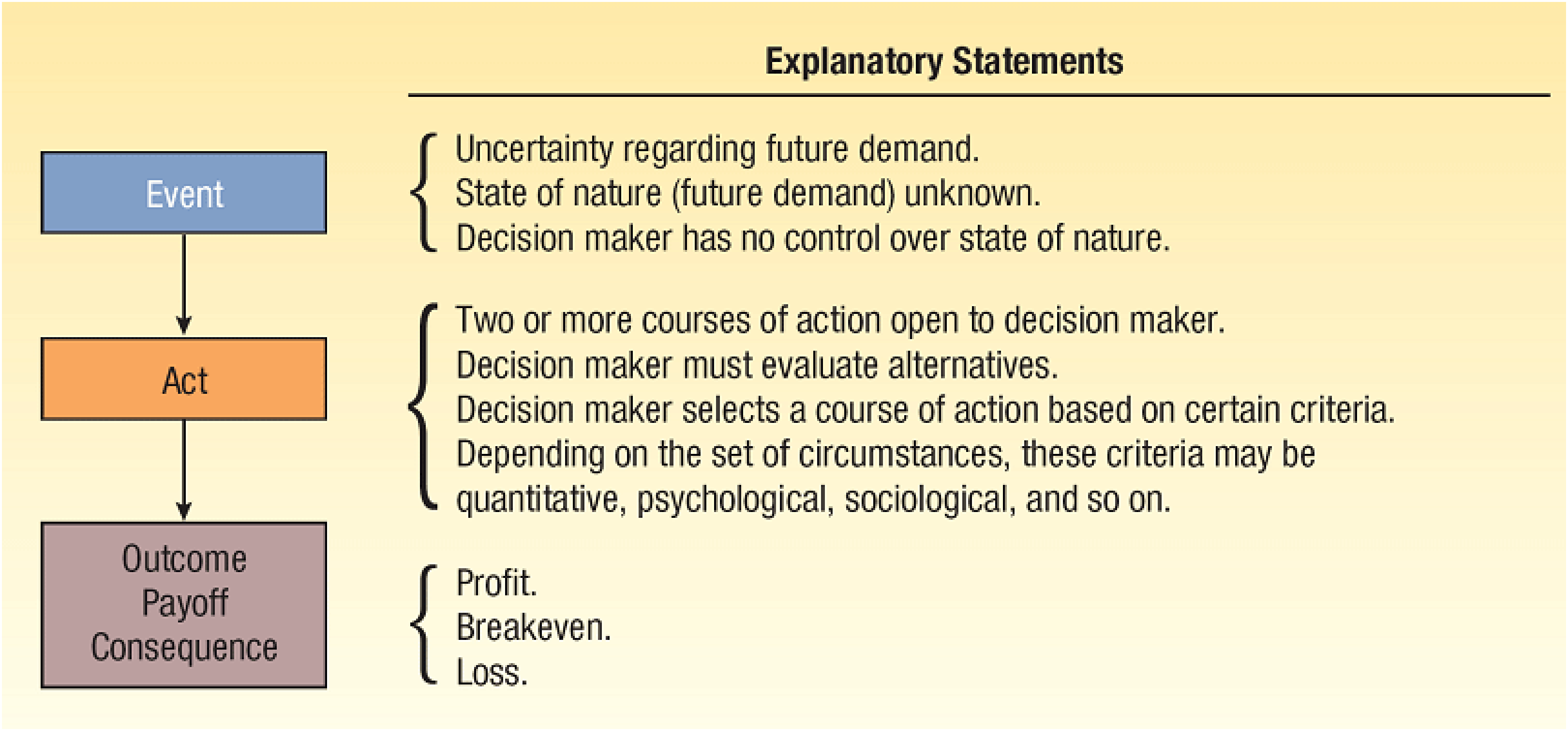
- Classical statistics focuses on estimating a parameter, such as the population mean, constructing confidence intervals, or hypothesis testing.
- Statistical Decision Theory (Bayesian statistics) is concerned with determining which decision, from a set of possible decisions, is optimal.

Elements of a Decision

There are three components to any decision-making situation:

- The available choices (alternatives or acts).
- The states of nature, which are not under the control of the decision maker - uncontrollable future events.
- The payoffs - needed for each combination of decision alternative and state of nature.

Decision Making



Payoff Table and Expected Payoff

A Payoff Table is a listing of all possible combinations of decision alternatives and states of nature.

The Expected Payoff or the Expected Monetary Value (*EMV*) is the expected value for each decision.

Calculating the EMV

$$EMV(A_i) = \sum [P(S_j) \cdot V(A_i, S_j)]$$

- Let A_i be the i^{th} decision alternative.
- Let $P(S_j)$ be the probability of the j^{th} state of nature.
- Let $V(A_i, S_j)$ be the value of the payoff for the combination of decision alternative A_i and state of nature S_j.
- Let $EMV(A_i)$ be the expected monetary value for the decision alternative A_i.

Decision Making Under Conditions of Uncertainty - Example

Bob Hill, a small investor, has $1,100 to invest. He has studied several common stocks and narrowed his choices to three, namely, Kayser Chemicals, Rim Homes, and Texas Electronics. He estimated that, if his $1,100 were invested in Kayser Chemicals and a strong bull market developed by the end of the year (that is, stock prices increased drastically), the value of his Kayser stock would more than double, to $2,400. However, if there were a bear market (i.e., stock prices declined), the value of his Kayser stock could conceivably drop to $1,000 by the end of the year. His predictions regarding the value of his $1,100 investment for the three stocks for a bull market and for a bear market are shown below. A study of historical records revealed that during the past 10 years stock market prices increased six times and declined only four times. According to this information, the probability of a market rise is .60 and the probability of a market decline is .40.

Purchase	Bull Market, S_1	Bear Market, S_2
Kayser Chemicals (A_1)	$2,400	$1,000
Rim Homes (A_2)	2,200	1,100
Texas Electronics (A_3)	1,900	1,150

EMV- Example

Purchase	Bull Market, S_1 (.60)	Bear Market, S_2 (.40)	Expected Payoff
Kayser Chemicals (A_1)	$2,400	$1,000	$1,840
Rim Homes (A_2)	2,200	1,100	1,760
Texas Electronics (A_3)	1,900	1,150	1,600

(*A*1)=(.6)($2,400)+(.4)($1,000) =$1,840
(A2)=(.6)($2,400)+(.4)($1,000) =$1,760
(A3)=(.6)($2,400)+(.4)($1,000) =$1,600

Opportunity Loss

Opportunity Loss or Regret is the loss because the exact state of nature is not known at the time a decision is made.

- The opportunity loss is computed by taking the difference between the optimal decision for each state of nature and the other decision alternatives.

Expected Opportunity Loss

EXPECTED OPPORTUNITY LOSS $\text{EOL}(A_i) = \Sigma\, [P(S_j) \times R(A_i, S_j)]$

where

$\text{EOL}(A_i)$ refers to the expected opportunity loss for a particular decision alternative.

$P(S_j)$ refers to the probability associated with the states of nature j.

$R(A_i, S_j)$ refers to the regret or loss for a particular combination of a state of nature and a decision alternative.

Opportunity Loss - Example

Purchase	Bull Market, S_1	Bear Market, S_2
Kayser Chemicals (A_1)	$2,400	$1,000
Rim Homes (A_2)	2,200	1,100
Texas Electronics (A_3)	1,900	1,150

	Opportunity Loss	
Purchase	Market Rise	Market Decline
Kayser Chemicals	$ 0	$150
Rim Homes	200	50
Texas Electronics	500	0

Opportunity Loss when Market Rises

Kayser:
$2,400 - $2,400= $0

Rim Homes:
$2,400 - $2,200 = $200

Texas Electronics:
$2,400 - $1,900 = $500

Opportunity Loss when Market Declines

Kayser:
$1,150 - $1,000= $150

Rim Homes:
$1,150 - $1,100 = $50

Texas Electronics:
$1,150 - $1,150 = $0

Expected Opportunity Loss

EXPECTED OPPORTUNITY LOSS $EOL(A_i) = \Sigma\, [P(S_j) \times R(A_i, S_j)]$

Purchase	0.60 Market Rise (Opportunity Loss)	0.40 Market Decline (Opportunity Loss)	Expected Opportunity Loss
Kayser Chemicals	$ 0	$150	$ 60
Rim Homes	200	50	140
Texas Electronics	500	0	300

(*A*1)=(.6)($0)+(.4)($150) =$60
(A2)=(.6)($200)+(.4)($50) =$140
(A3)=(.6)($500)+(.4)($0) =$300

Maximin, Maximax, and Minimax Regret Strategies

Payoff Table

Purchase	Bull Market, S_1	Bear Market, S_2	Maximin	Maximax
Kayser Chemicals (A_1)	$2,400	$1,000	1,000	2,400
Rim Homes (A_2)	2,200	1,100	1,100	2,200
Texas Electronics (A_3)	1,900	1,150	1,150	1,900

Opportunity Loss Table

Purchase	Opportunity Loss		Minimax Regret
	Market Rise	Market Decline	
Kayser Chemicals	$ 0	$150	150
Rim Homes	200	50	200
Texas Electronics	500	0	500

Maximin, Maximax, and Minimax Regret Strategies

Maximin strategy maximizes the minimum gain. It is a pessimistic strategy.

Maximax strategy maximizes the maximum gain. Opposite of a maximin approach, it is an optimistic strategy

Minimax regret strategy minimizes the maximum regret (opportunity loss). This is another pessimistic strategy

Value of Perfect Information

What is the worth of information known in advance before a strategy is employed?

Expected Value of Perfect Information (*EVPI*) is the difference between the expected payoff if the state of nature were known and the optimal decision under the conditions of uncertainty.

EVPI Example

EVPI = Expected value under conditions of certainty
− Expected value under conditions of uncertainty

Step 1: Compute the Expected Value Under Certainty

State of Nature	Decision	Payoff	Probability of State of Nature	Expected Payoff
Market rise, S_1	Buy Kayser	$2,400	.60	$1,440
Market decline, S_2	Buy Texas Electronics	1,150	.40	460
				$1,900

Expected Value Under Certainty

EVPI Example

Step 2: Compute the Expected Value Under Uncertainty

Purchase	Bull Market, S_1 (.60)	Bear Market, S_2 (.40)	Expected Payoff
Kayser Chemicals (A_1)	$2,400	$1,000	$1,840
Rim Homes (A_2)	2,200	1,100	1,760
Texas Electronics (A_3)	1,900	1,150	1,600

Step 3: Subtract the Expected Value Under Uncertainty from the Expected Value Under Certainty

$1,900	Expected value of stock purchased under conditions of certainty
−1,840	Expected value of purchase (Kayser) under conditions of uncertainty
$ 60	Expected value of perfect information

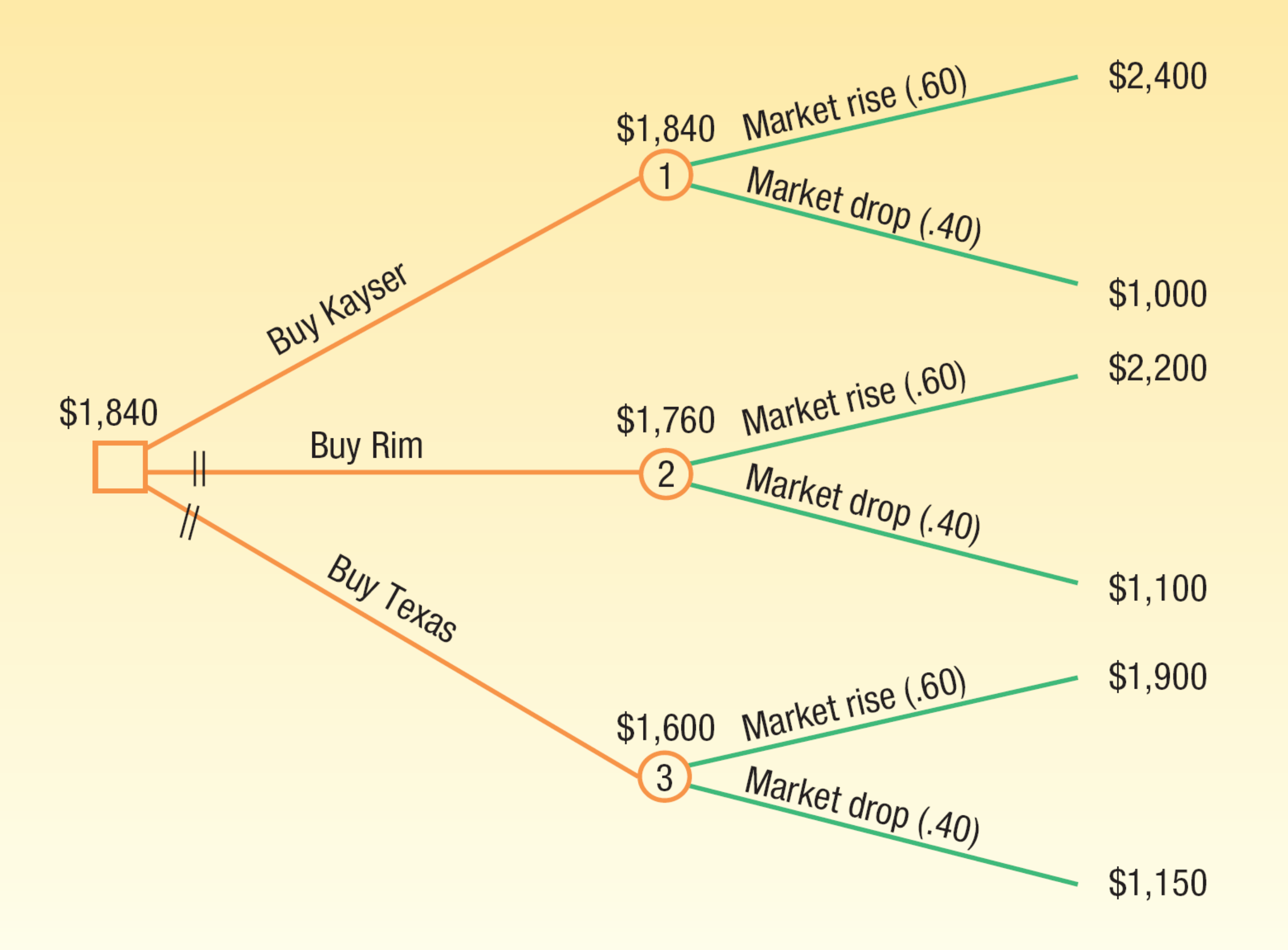
Sensitivity Analysis and Decision Trees

- Sensitivity Analysis examines the effects of various probabilities for the states of nature on the expected values for the decision alternatives.
- Decision Trees are useful for structuring the various alternatives. They present a picture of the various courses of action and the possible states of nature.

Decision Tree

A **decision tree** is a picture of all the possible courses of action and the consequent possible outcomes.

- A box is used to indicate the point at which a decision must be made,
- The branches going out from the box indicate the alternatives under consideration


$1,840
Buy Kayser
Buy Rim
Buy Texas
$1,840
1
Market rise (.60)
Market drop (.40)
$2,400
$1,000
$1,760
2
Market rise (.60)
Market drop (.40)
$2,200
$1,100
$1,600
3
Market rise (.60)
Market drop (.40)
$1,900
$1,150

End of Chapter 20